ELSEVIER

Journal of Financial Markets 6 (2003) 607–624

Journal of FINANCIAL MARKETS

www.elsevier.com/locate/econbase

Market structure and diversification of mutual funds ☆

Oz Shy[a], Rune Stenbacka[b,*]

[a] *University of Haifa, Israel, and Swedish School of Economics, Finland*

[b] *Swedish School of Economics, Helsinki, Finland, and Academy of Finland, P.O. Box 479, 00101 Helsinki, Finland*

Abstract

This study characterizes a systematic relationship between the diversification incentives and the market structure of the mutual funds industry with investors differentiated by their attitude towards risk. With sufficiently low competition the subgame perfect portfolio equilibrium exhibits maximal risk differentiation. With intensified competition intermediate funds, i.e., those attracting investors with intermediate attitudes towards risk, select diversified portfolios. Finally, we offer a general characterization of how imperfect competition between risk-differentiated funds will generate an equilibrium relationship between risk and expected returns. © 2002 Elsevier B.V. All rights reserved.

JEL classification: G11; G12; G21

Keywords: Diversification of mutual funds; Risk taking; Market structure; Mutual funds competition

1. Introduction

To an increasing extent investors do not invest in individual securities one at a time. Instead investors want to combine many assets into well-diversified portfolios in order to reduce the risk of their overall investment. Institutions

☆ We thank an anonymous referee for constructive comments and suggestions.

*Corresponding author. Tel.: 00358-943133433; fax: 00358-943133382.

E-mail address: rune.stenbacka@shh.fi (R. Stenbacka).

PII: S1386-4181(01)00028-3

like mutual funds offer such services. By the end of 1999 mutual funds were estimated to manage assets in excess of those at commercial banks in the U.S. Overall equity mutual funds represented 18 per cent of all U.S. equity, by value. However, mutual funds are differentiated along several dimensions as they offer portfolios which are diversified according to different characteristics, thereby forming different market categories. In fact, the number of mutual funds in the U.S. exceeds the number of stocks traded on both the NYSE and the AMEX added together, reaching far beyond 6000 funds. Similarly, in this industry the degree of segmentation has increased, reportedly reaching around 41 different categories (see, Massa, 2000). Parallel to the expansion of the mutual fund industry the financial markets have been transformed by a frantic pace of financial innovation directed towards security design (see, for example, Allen and Gale, 1994). In light of this evidence we can conclude that diversification and differentiation represent crucial strategic instruments whereby fund managers compete for investors.

In this paper we design a stylized formal model of the mutual funds industry in order to investigate the structural relationship between market structure and the incentives of mutual funds to diversify their asset portfolios. In general, the way managers of mutual funds construct their asset portfolios will affect the risk inferences drawn by investors and thereby the expected portfolio return required by these investors. In fact, the choice of riskiness incorporated in the portfolio selection might constitute an important strategic instrument with respect to the competition taking place among mutual funds. In this respect each fund manager faces a number of central strategic decisions: Should the fund concentrate on a few industries or a few geographical regions (countries) so as to exploit gains from specialization based on economics of scale with respect to, for example, monitoring as a way to create a competitive advantage for itself? Or, should the fund invest in a portfolio with maximal diversification so as to minimize the risk of its asset portfolio thereby attracting risk averse or liquidity-oriented investors who will accept a lower expected rate of return?

In the present paper we build a formal model of strategic competition between mutual funds in order to analyze this tradeoff between specialization and diversification. In particular, our model makes it possible to provide answers for the following questions: How will competition between mutual funds impact on their diversification decisions? Is there a systematic relationship between the diversification incentives and market structure in the mutual funds industry?

The research literature devoted to the mutual funds industry has so far presented no formal models of competition between funds with diversification as the strategic instrument in an oligopolistic sense. Nanda et al. (2000) focus on a perfectly competitive mutual fund industry with risk-neutral investors and heterogeneous managerial ability. Within such a context they demonstrate how funds differentiated by load types can separate investors with different liquidity needs. Earlier Chordia (1996) explored a monopolistic industry and demonstrated how a monopoly fund may enhance investors' welfare. This happens because the fund is able to lower liquidity costs by diversifying across investors who face less than perfectly correlated liquidity shocks. In contrast to these contributions the present study offers a general characterization of how imperfect competition between funds generates an equilibrium relationship between risk and expected returns in the absence of heterogeneity in terms of manager abilities.

An important branch of the literature on mutual fund has focused on the explanations for why we observe intertemporal persistence in the performance of the funds. For example, Khorana and Nelling (1998) offer empirical evidence of this type of persistence. Grinblatt et al. (1995) reported that 77 percent of the mutual funds were "momentum investors", operating with a strategy of buying stock that were past winners. These authors then examined the extent to which herding and momentum investing affect the performance of the funds. Another body of analytical work has focused on the issue of what service mutual funds offer so as to motivate investors into these funds. One explanation refers to fund managers as being better informed than investors and within such an agency approach it is possible to characterize optimal compensation contracts so as to induce the fund managers to expend effort to generate information and to use it appropriately for the portfolio selections (see, for example, Dybvig et al., 1999).

In contrast to the mutual fund industry a number of studies have explored the relationship between the market structure and the incentives of risk taking, in a more general sense than that of diversification exclusively, in another financial industry, namely the banking industry. Matutes and Vives (2000) have examined the consequences of imperfect competition for deposits on the risk-taking incentives of banks with a particular focus of the policy issues relevant for the banking industry. Koskela and Stenbacka (2000) focus on the relationship between market structure and risk taking in lending markets where the borrowers have access to mean-shifting investment technologies and they ask whether there is a tradeoff between competition and financial fragility. Still other studies like Broecker (1990), Gehrig (1998) and Shaffer (1998) have investigated the consequences of adverse selection resulting from the unobserved characteristics of borrowers.

In the present study we construct a model of a differentiated mutual funds industry in which the funds are engaged in two-stage competition based on (i) how to diversify their asset portfolios and (ii) what fees to impose on the investors. Our study characterizes a systematic relationship between the diversification incentives and the market structure of mutual funds. With sufficiently low competition, i.e. in a duopoly, the subgame perfect portfolio equilibrium will exhibit maximal risk differentiation and the funds will choose a strategy of specialization. With intensified competition we find that intermediate funds, i.e. those attracting investors with intermediate attitudes towards risk, will select diversified portfolios.

The article is organized as follows. Section 2 presents a model of a market for competition between two mutual funds. Section 3 establishes a general principle of maximal risk differentiation according to which the subgame perfect portfolio exhibits asset specialization. In Section 4 the competition is intensified through the introduction of a third fund and we demonstrate that the fund attracting investors with intermediate attitudes towards risk will select a perfectly diversified portfolio. Section 5 presents a characterization of how imperfect competition between funds differentiated by risk will generate an equilibrium relationship between risk and expected returns. Finally, Section 6 offers some concluding comments.

2. A model of the market for mutual funds

We initially consider an economy with two profit-maximizing mutual funds, called fund 1 and fund 2. There is a continuum of investors, each with one unit of currency (say \$1) to be invested into one (and only one) of the funds. Appendix A demonstrates that risk-neutral investors indeed choose only one fund even if they are allowed to diversify across all available funds.

2.1. The funds

Funds compete for money by selecting an investment policy. We denote by r_i the return on fund i, $i = 1, 2$. Mutual funds subsequently invest the funds acquired in assets which can be interpreted as business projects, stocks, or government bonds. In the long run each fund commits itself to an irreversible portfolio policy, which is announced to the investors. In particular, this investment policy will determine the riskiness incorporated in the fund's asset portfolio.

When funds compete with respect to their choices of portfolio riskiness they must anticipate the effects of their portfolio choice on the resulting fee they can charge in equilibrium. To capture such effects we apply subgame perfectness as the equilibrium concept for funds' diversification decisions.

We capture the diversification choice of funds within the framework of a very simple stylized model. Each mutual fund can diversify each dollar invested into the two assets, denoted by A and B. The gross return on a \$1 investment in asset A is stochastic and is given by

$$R_A = \begin{cases} \alpha & \text{with probability } \theta, \\ 0 & \text{with probability } 1 - \theta. \end{cases} \tag{1}$$

With probability θ the risky asset yields the favorable return α, whereas with the complementary probability $1-\theta$ the fund loses its entire investment in asset A. In contrast, asset B is safe yielding a certain gross per-dollar return of $\beta > 1$, and therefore can be viewed as government bonds. We make the following assumption.

Assumption 1. An investment which is liquidated after one period can be recovered, but it generates a zero rate of return. The gross expected return on the risky asset exceeds that of the safe asset. Formally, $\theta\alpha > \beta$.

The rate of return on both types of assets is realized only if the investment is kept for two periods.

Let m_i $(0 \leqslant m_i \leqslant 1)$ denote the fraction invested in the risky asset (asset A), $i = 1, 2$, so that $1 - m_i$ is the fraction invested into the safe asset (asset B). Hence, the expected return of fund i is given by $m_i\theta\alpha + (1 - m_i)\beta$.

Each fund levies a fee of f_i per dollar invested, $i = 1, 2$. Then, if n_i denotes the (endogenously determined) number of consumers investing in fund i, the profit of fund i is $\pi_i = f_i n_i$, $i = 1, 2$. In addition, the fund charges an amount of c $(0 \leqslant c < 1)$ if investors liquidate their investment after one period only. This penalty is proportional to the fund's holding of the risky asset (see (2) below). This cost (borne by the investor who liquidates at $t = 1$) may reflect borrowing costs incurred by a fund in order to match the withdrawal of investments into illiquid assets.

2.2. Timing and sequence of decisions

The timing of the decisions by the funds and the investors is composed of five stages as illustrated in Table 1.

Table 1 assumes a three-stage game where in the first stage ($t = -2$) the funds construct their funds by committing to the degree of diversification between the safe and their risky investments. In the second stage ($t = -1$), each fund sets its fee knowing the funds' characteristics and the fee set by the

Table 1
Timing and sequence of decisions

Player(s)	Period	Actions taken
Funds	$t = -2$	Funds announce their diversification parameters (m_1 and m_2)
Funds	$t = -1$	Funds announce their fees (f_1 and f_2)
Investors	$t = 0$	Investors choose which fund to invest in
Nature	$t = 1$	Investors may face liquidity needs and withdraw their investments
Nature	$t = 2$	Investment returns realized, funds pay interest plus principal

competing fund. In the third stage ($t = 0$), each investor selects the fund that maximizes his expected return, to be defined below. We will be solving for a subgame-perfect equilibrium for this game.

2.3. Investors

We focus on heterogeneous investors (consumers) who vary in their attitudes towards risk in the sense that they face differentiated liquidity needs. The investors are uniformly distributed with a uniform density on the unit interval $[0, 1]$ in accordance to increased probability of realizing a liquidity need in period $t = 1$, prior to the realization of the return on the funds. With uniform density, we normalize the total mass of investors to equal $\eta > 0$. Thus, an investor indexed by $\lambda = 0$ will not realize a liquidity need in $t = 1$, whereas an investor indexed by $\lambda = 1$ will surely experience a liquidity need and will therefore withdraw his investment from his selected fund.

In order to reduce the amount of notation, we assume that investors do not discount future incomes. Then, the expected return on maintaining a one-dollar investment in fund i, $i = 1, 2$, in periods 1 and 2, is $(1 - m_i)\beta + m_i[\theta\alpha + (1 - \theta) \cdot 0]$. If, however, this investor experiences a liquidity need in $t = 1$ and withdraws from the fund, the investor receives an amount of $1 - m_i + m_i(1 - c)$ since $1 - m_i$ is invested in the safe liquid asset, whereas m_i is invested in the risky asset, in which case the investor pays a penalty if c for early withdrawal. Altogether, the expected return to an investor indexed by λ who invests his \$1 in fund i, $i = 1, 2$ is

$$\mathrm{E}_\lambda R_i = \lambda[1 - m_i + m_i(1 - c)] + (1 - \lambda)[(1 - m_i)\beta + m_i\theta\alpha] - f_i. \tag{2}$$

The first term in (2) reflects the amount an investor liquidates when a liquidity need is realized, which happens with probability λ. The second term is the fund's return in an event that the investor does not liquidate at $t = 1$.

3. Equilibrium diversification and fee determination

Table 1 defines a three-stage game between the funds competing on investors. We now solve for the subgame-perfect equilibrium for this game by solving it backwards starting with investors' selection of funds. Then, we will concentrate on the stage of funds' fee competition taking the portfolio investment allocations as given. Finally, we solve for the equilibrium portfolio selections of the two mutual funds.

3.1. Stage III ($t = 0$): investors' selection of funds

In what follows, we analyze investors who choose to save their \$1 in one and only one fund. Appendix A generalizes this model by allowing investors to diversify their investment between the funds and demonstrates that risk-neutral investors will always end up investing in only one fund.

Suppose, with no loss of generality, that $m_2 > m_1$, meaning that the investment portfolio of fund 2 has a higher concentration on the risky asset (asset A) than the portfolio of fund 1.

The probability of realizing a $t = 1$ liquidity need of an investor $\hat{\lambda}$ who is indifferent between fund 1 with a portfolio weight m_1 on the risky asset A and fund 2 with a portfolio weight m_2 on asset B is determined from (2) by

$$\begin{aligned}\mathrm{E}_{\hat{\lambda}}R_1 &= \hat{\lambda}[1 - m_1 + m_1(1-c)] + (1-\hat{\lambda})[(1-m_1)\beta + m_1\theta\alpha] - f_1 \\ &= \hat{\lambda}[1 - m_2 + m_2(1-c)] + (1-\hat{\lambda})[(1-m_2)\beta + m_2\theta\alpha] - f_2 = \mathrm{E}_{\hat{\lambda}}R_2.\end{aligned}$$

$$\hat{\lambda} = \frac{(m_2 - m_1)\theta\alpha - (m_2 - m_1)\beta + f_1 - f_2}{(m_2 - m_1)(\theta\alpha + c - \beta)}. \tag{3}$$

Consequently, all investors indexed by $0 \leqslant \lambda \leqslant \hat{\lambda}$ invest in fund 2 (the more risky fund) and all investors indexed by $\hat{\lambda} < \lambda \leqslant 1$ invest in fund 1 (the less risky fund). Clearly, from (3) we can see that the market share of fund 2 declines when the fund increases its per-dollar invested fee, f_2, and increases when the rival fund raised its fee, f_1. The market allocation of investors between the two funds is illustrated in Fig. 1.

3.2. Stage II ($t = -1$): competition in fees

Fund 1 treats the portfolio choices, m_1 and m_2, and the fee f_2 as given and chooses its fee f_1 that solves

$$\max_{f_1} \pi_1 = (1-\lambda)\eta f_1 = \left[1 - \frac{(m_2 - m_1)\theta\alpha - (m_2 - m_1)\beta + f_1 - f_2}{(m_2 - m_1)(\theta\alpha + c - \beta)}\right]\eta f_1. \tag{4}$$

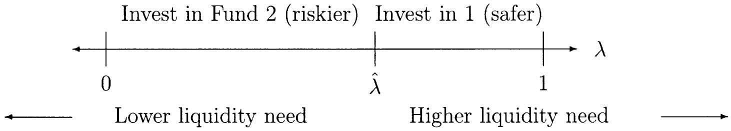

Fig. 1. Allocation of investors between the two mutual funds. *Remark*. The condition under which $\hat{\lambda} \geqslant \frac{1}{2}$ is given in Proposition 2 below.

Assumption 1 implies that the profit function (4) is strictly concave with respect to f_1, implying that there exists a unique and well-defined best response function given by

$$f_1 = \frac{(m_2 - m_1)c + f_2}{2}. \tag{5}$$

Similarly, mutual fund 2 treats the portfolio choices, λ_1 and λ_2, and the fee f_1 as given and chooses its fee f_2 that solves

$$\max_{f_2} \pi_2 = \lambda\eta f_2 = \left[\frac{(m_2 - m_1)\theta\alpha - (m_2 - m_1)\beta + f_1 - f_2}{(m_2 - m_1)(\theta\alpha + c - \beta)}\right]\eta f_2, \tag{6}$$

yielding the best-response function given by

$$f_2 = \frac{(m_2 - m_1)\theta\alpha - (m_2 - m_1)\beta + f_1}{2}. \tag{7}$$

Eqs. (5) and (7) imply that the funds' fee decisions are strategic complements as a rise in the fee of one of the funds implies an increase in the fee of the competing fund. Solving the system of equations determined by (5) and (7) yields

$$f_1 = \frac{(m_2 - m_1)(\theta\alpha - \beta + 2c)}{3} \quad \text{and} \quad f_2 = \frac{(m_2 - m_1)(2\theta\alpha - 2\beta + c)}{3}. \tag{8}$$

From (8) we can immediately conclude that the Nash equilibrium fees are proportional to the degree of risk differentiation $m_2 - m_1$. Substituting (8) into the funds' profit functions (4) and (6) yields

$$\pi_1 = \frac{\eta(m_2 - m_1)(\theta\alpha - \beta + 2c)^2}{9(\theta\alpha - \beta + c)} \quad \text{and} \quad \pi_2 = \frac{\eta(m_2 - m_1)(2\theta\alpha - 2\beta + c)^2}{9(\theta\alpha - \beta + c)}. \tag{9}$$

From (9) we can infer that the equilibrium profits of the two competing funds are increasing in the degree of risk differentiation between the two funds. In the next subsection we will turn to an analysis of the portfolio selections of the two competing funds.

3.3. Stage I ($t = -2$): portfolio choice

In the first stage mutual fund 1 takes the portfolio choice of fund 2, $m_2 \leqslant 1$, as given and decides on its investment portfolio, m_1, to maximize π_1 given in (9). Similarly, for a given $m_1 \geqslant 0$ fund 2 chooses m_2 to maximize π_2 also given in (9). We say that portfolios exhibit *maximal risk differentiation* if $|m_2 - m_1| = 1$.

Proposition 1. With two competing funds, the subgame perfect portfolio equilibrium exhibits maximal risk differentiation. Formally, $\langle m_1, m_2 \rangle = \langle 0, 1 \rangle$ *is a subgame-perfect equilibria.*[1]

Proof. The profit functions (9) imply that π_1 is maximized at $m_1 = 0$ given that $m_2 = 1$. Analogously π_2 is maximized at $m_2 = 1$ given that $m_1 = 0$. We are left to prove that there are no interior subgame-perfect equilibria. Suppose there exists an equilibrium where $0 < m_1 \leqslant m_2 \leqslant 1$. Then, it is immediate from (9) that π_1 can be increased by the portfolio choice $\hat{m}_1 = m_1 - \varepsilon$, where $0 < \varepsilon < m_1$. Therefore, an interior equilibrium does not exist. □

Proposition 1 expresses that the two competing funds will choose to completely specialize their investment portfolios. This means that the funds engage in risk differentiation in order to exploit the different liquidity needs of investors.

Intuitively, by maximal risk differentiation the funds are able to reduce the intensity of the subsequent stage of competition in fees. In fact, as (8) makes clear, the Nash equilibrium incorporates fees which are linearly increasing in the risk differential between the two funds.

3.4. Equilibrium fees, market shares, and profits

Substituting (8) into the probability of a liquidity need, (3), for an investor indifferent between the risky and riskless fund yields

$$\hat{\lambda} = \frac{2\theta\alpha - 2\beta + c}{3(\theta\alpha - \beta + c)}. \tag{10}$$

Assumption 1 ensures that $0 < \hat{\lambda} < 1$. Therefore,

Proposition 2. The market share of the risky fund (fund 2) is larger than that of the risk-free fund (fund 1) if and only if the expected return on the risky asset net of the early-withdrawal cost exceeds the return on the safe asset. Formally, $\hat{\lambda} \geqslant \frac{1}{2}$ *if and only if* $\theta\alpha - c \geqslant \beta$.

Proposition 2 implies that the market share is an increasing function of the fund's riskiness. It also implies that the population of investors is equally divided between the two funds if and only if both funds yield the same expected return net of liquidation costs. Since the safe asset is liquid, the investors subtract the cost of early withdrawal c from the expected return of the risky asset only. Thus, since an investor indexed by $\lambda = \frac{1}{2}$ has a 50 percent

[1] Clearly, by relabeling fund 1 as fund 2 and fund 2 as fund 1, and reworking all the calculations accordingly, $\langle m_2, m_1 \rangle = \langle 0, 1 \rangle$ becomes a subgame perfect portfolio equilibrium.

probability of facing a liquidity shock, such an investor compares the expected return net of withdrawal cost of the two funds.

Substituting $m_1 = 0$ and $m_2 = 1$ into (8), the subgame perfect equilibrium fees are

$$f_1 = \frac{\theta\alpha - \beta + 2c}{3} \quad \text{and} \quad f_2 = \frac{2\theta\alpha - 2\beta + c}{3}. \tag{11}$$

Substituting $m_1 = 1$ and $m_2 = 0$ into (9),

$$\pi_1 = \frac{\eta(\theta\alpha - \beta + 2c)^2}{9(\theta\alpha - \beta + c)} \quad \text{and} \quad \pi_2 = \frac{\eta(2\theta\alpha - 2\beta + c)^2}{9(\theta\alpha - \beta + c)}. \tag{12}$$

Assumption 1 ensures that $f_i, \pi_i > 0,\ i = 1, 2$. From the equilibrium fees (11) and the associated profits (12) we can conclude the following.

Proposition 3. The risky fund (*fund* 2) *charges a higher fee and earns a higher profit compared with the risk-free fund* (*fund* 1) *if and only if the expected return on the risky asset net of the early-withdrawal cost exceeds the return on the safe asset. Formally,* $f_2 \geqslant f_1$ *and* $\pi_2 \geqslant \pi_1$ *if and only if* $\theta\alpha - c \geqslant \beta$.

Proposition 3 implies that the equilibrium fees and profits are increasing in the riskiness of the fund if and only if the expected return on the risky asset net of the early-withdrawal cost exceeds the return on the safe asset. Notice that the condition (and therefore the intuition) of Proposition 3 is identical to that of Proposition 2. Thus, a higher expected return of the risky fund net of an early-withdrawal cost induces fund 2 to charge a higher fee. With a larger market share, fund 2 must earn a higher profit. Additional features of this equilibrium are listed in the following proposition.

Proposition 4. (a) *The equilibrium fee charged by each mutual fund, and the associated profits increase with the expected return of the risky asset. Furthermore, these increases are larger for the risky fund. Formally,*

$$\frac{\mathrm{d}f_2}{\mathrm{d}\alpha} > \frac{\mathrm{d}f_1}{\mathrm{d}\alpha} > 0 \quad \textit{and} \quad \frac{\mathrm{d}\pi_2}{\mathrm{d}\alpha} > \frac{\mathrm{d}\pi_1}{\mathrm{d}\alpha} > 0.$$

(b) *The equilibrium fee charged by each mutual fund, and the resulting profit levels decline with the return on the safe asset. Furthermore, these declines are stronger for the risky fund. Formally,*

$$\frac{\mathrm{d}f_2}{\mathrm{d}\beta} < \frac{\mathrm{d}f_1}{\mathrm{d}\beta} < 0 \quad \textit{and} \quad \frac{\mathrm{d}\pi_2}{\mathrm{d}\beta} < \frac{\mathrm{d}\pi_1}{\mathrm{d}\beta} < 0.$$

Proposition 4 reveals that the equilibrium response to changes in the returns of the underlying assets is stronger in the risky fund than in the risk-free fund. Part (b) follows from Assumption 1 which implies that competition between the funds intensifies as β gets closer to $\theta\alpha$. Consequently, in this case both equilibrium fees and profits fall.

To conclude this section, we have explored an important insight regarding the relationship between the diversification equilibrium and the market structure of the mutual fund industry. Proposition 1 highlights that complete specialization (no diversification) weakens fee competition and therefore maximizes profits in an industry with only two mutual funds. Thus, with two competing funds the model predicts maximal risk differentiation and no portfolio diversification in equilibrium. In order to generate portfolio diversification as an equilibrium outcome we need a market structure with more intense competition than that of a duopoly. Consequently, our model predicts that diversification can be observed only in markets with more than two mutual funds. In the next section we explore the consequences of intensified competition between funds in greater detail.

4. Intensified competition among funds

Suppose now that there are three mutual funds, each of which has access to the two investment assets described in the previous section. We now search for an equilibrium in which the three funds will divide the market according to investors' liquidity needs as illustrated in Fig. 2.

Thus, with no loss of generality, we model fund 3 as the most risky (high concentration on asset A), fund 2 as partially risky, and fund 1 as the least risky (highest concentration on asset B).

Our analysis of the two-fund investment market, as summarized in (9), has shown that mutual funds can increase their clientele and profit by completely specializing their investment portfolios in either the risky asset or the safe asset. Therefore, without repeating the analysis, the same argument implies that $m_1 = 0$ and $m_3 = 1$.

4.1. Stage III ($t = 0$): investors' selection of funds

Based on the expected return of each fund, (2), and in view of Fig. 2, the probability of a liquidity need of an investor who is indifferent between fund 3

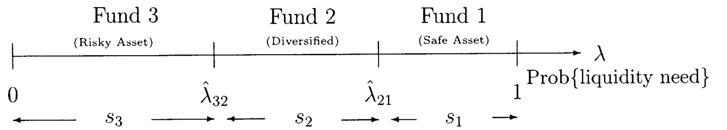

Fig. 2. Investors' allocation among the three mutual funds. *Remark*. s_i is the market share of fund i [Proposition 5(b) below].

and fund 2, as well as that of an investor who is indifferent between fund 2 and fund 1 are given by

$$\hat{\lambda}_{32} = \frac{(1-m_2)(\theta\alpha-\beta)+f_2-f_3}{(1-m_2)(\theta\alpha-\beta+c)} \quad \text{and} \quad \hat{\lambda}_{21} = \frac{m_2(\theta\alpha-\beta)+f_1-f_2}{m_2(\theta\alpha-\beta+c)}, \tag{13}$$

respectively. Therefore, the profit function of mutual fund 1 is given by

$$\pi_1 = (1-\lambda_{21})\eta f_1 = \left[1 - \frac{m_2(\theta\alpha-\beta)+f_1-f_2}{m_2(\theta\alpha-\beta+c)}\right]\eta f_1, \tag{14}$$

whereas that of mutual fund 3 is given by

$$\pi_3 = \lambda_{32}\eta f_3 = \left[\frac{(1-m_2)(\theta\alpha-\beta)+f_2-f_3}{(1-m_2)(\theta\alpha-\beta+c)}\right]\eta f_3. \tag{15}$$

The profit of the intermediate fund is given by

$$\pi_2=(\lambda_{21}-\lambda_{32})\eta f_2=\left[\frac{m_2(\theta\alpha-\beta)+f_1-f_2}{m_2(\theta\alpha-\beta+c)} - \frac{(1-m_2)(\theta\alpha-\beta)+f_2-f_3}{(1-m_2)(\theta\alpha-\beta+c)}\right]\eta f_2. \tag{16}$$

4.2. Stage II ($t = -1$): fee competition

We initially explore the equilibrium with respect to the stage of fee competition. From (14) and (16) it can be easily verified that each profit function π_i is strictly concave with respect to the fee charged by fund i, f_i. Therefore, simple optimization with respect to fees yields the following best-response functions:

$$f_1 = \frac{cm_2+f_2}{2}, \quad f_2 = \frac{(1-m_2)f_1+m_2 f_3}{2}, \text{ and } f_3 = \frac{(1-m_2)(\theta\alpha-\beta)+f_2}{2}. \tag{17}$$

By solving the system of equations defined by the three best-response functions (17) we find the three Nash equilibrium fees as functions of the investment diversification of fund 2. These equilibrium fees are given by

$$f_1 = \frac{m_2[(1-m_2)(\theta\alpha-\beta)-c(m_2-4)]}{6},\ f_2 = \frac{m_2(1-m_2)(\theta\alpha-\beta+c)}{3},$$

and

$$f_3 = \frac{(1-m_2)[(m_2+3)(\theta\alpha-\beta)+cm_2]}{6}. \tag{18}$$

These equilibrium fees are next substituted into the profit functions (14) and (16) in order to obtain the profit functions expressed in terms of the diversification decisions.

4.3. Stage I ($t = -2$): portfolio choice

As previously argued, fund 3 specializes its investment portfolio in the risky asset ($m_3 = 1$), whereas fund 1 specializes in the safe asset ($m_1 = 0$). Therefore, we need only find the degree of diversification of fund 2, which attracts investors belonging to the medium range in terms of their attitudes towards risk. Substituting for f_1, f_2, and f_3 into the profit function of fund 2 given in (16) yields

$$\pi_2 = \frac{\eta m_2(1 - m_2)(\theta\alpha - \beta + c)}{9}.$$

Differentiating with respect to the portfolio choice variable, m_2, yields the necessary first-order condition

$$0 = \frac{\mathrm{d}\pi_2}{\mathrm{d}m_2} = \frac{\eta(1 - 2m_2)(\theta\alpha - \beta + c)}{9},$$

from which we conclude that $m_2 = 1/2$. Consequently, the intermediate fund will perfectly diversify its portfolio across the two available assets by allocating 50 percent of the portfolio to the risky asset and 50 percent to the safe asset.

4.4. Equilibrium fees, market shares, and profits

We are now able to precisely characterize the subgame perfect diversification decisions of an industry with three funds. Funds 1 and 3 specialize their portfolios by investing exclusively into the riskless and risky asset, respectively. In contrast to these funds, the portfolio equilibrium is characterized by the intermediate fund diversifying by splitting its resources on an equal basis between the two available assets. It is remarkable that this equilibrium portfolio is invariant to changes in the returns of assets A and B as long as the expected return of A exceeds that of B.

Substituting $m_2 = 1/2$ into (18) we obtain

$$f_1 = \frac{\theta\alpha - \beta + 7c}{24}, \quad f_2 = \frac{\theta\alpha - \beta + c}{12}, \quad \text{and} \quad f_3 = \frac{7\theta\alpha - 7\beta + c}{24}. \tag{19}$$

Substituting $m_2 = 1/2$ and (19) into (14) and (16) yield the equilibrium profit of each mutual fund. Thus,

$$\pi_1 = \frac{\eta(\theta\alpha - \beta + 7c)^2}{288(\theta\alpha - \beta + c)}, \quad \pi_2 = \frac{\eta(\theta\alpha - \beta + c)}{36} \quad \text{and} \quad \pi_3 = \frac{\eta(7\theta\alpha - 7\beta + c)^2}{288(\theta\alpha - \beta + c)}. \tag{20}$$

Finally, let s_i ($0 \leqslant s_i \leqslant 1$) denote the market share of mutual fund i, $i = 1, 2, 3$. Substituting $m_2 = 1/2$ and (19) into (13) yields

$$s_1 = 1 - \hat{\lambda}_{21} = \frac{\theta\alpha - \beta + 7c}{12(\theta\alpha - \beta + c)}, \quad s_2 = \hat{\lambda}_{21} - \hat{\lambda}_{32} = \frac{1}{3}$$
$$\text{and } s_3 = \hat{\lambda}_{32} = \frac{7\theta\alpha - 7\beta + c}{12(\theta\alpha - \beta + c)}. \tag{21}$$

We can now characterize the portfolio configurations in the market with three mutual funds.

Proposition 5. (a) *In the subgame perfect portfolio equilibrium funds* 3 *and* 1 *specialize by concentrating exclusively on assets A and B, respectively, whereas the intermediate fund fully diversifies by allocating an equal amount of resources to the available assets.*

(b) *The riskiest fund attracts more consumers than the less risky funds if and only if the expected return net of early withdrawal cost of this fund exceeds the return on the safe fund. Formally,* $s_3 \geqslant s_2 \geqslant s_1$ *if and only if* $\theta\alpha - c \geqslant \beta$.

(c) *If* $\theta\alpha - c \geqslant \beta$, *the fund which specializes in investing in the risky asset charges the highest fee, that is,* $f_3 > \max\{f_1, f_2\}$. *However, the diversifying fund charges a higher fee than the safe fund if and only if the expected return on the risky asset exceeds that of the safe asset by a sufficient margin. Formally,* $f_2 \geqslant f_1$ *if and only if* $\theta\alpha - \beta \geqslant 5c$.

(d) *If* $\theta\alpha - c \geqslant \beta$, *the fund that specializes in investing in the risky asset earns a higher profit than the safe fund, that is,* $\pi_3 \geqslant \pi_1$.

What happens if competition within in the mutual fund industry is intensified due to entry of additional funds? In principle, we can easily generalize the analysis above to characterize the best-response functions defining the necessary conditions for a subgame perfect diversification equilibrium with an arbitrary number of funds in the industry. However, it is analytically very tedious to solve the system of recursive equations defined by these best-response functions. Nevertheless, as an earlier version of this paper indicated, we have strong reasons to conjecture that further entry into the fund industry will generate an equilibrium pattern of portfolio selection characterized by agglomeration towards low risk taking. Such an hypothesis could be made subject to empirical testing.

5. Equilibrium mean–variance relationship

Our model incorporates a simple, but yet very general, characterization of how imperfect competition between funds differentiated by risk will generate an equilibrium relationship between risk and expected returns of these funds. This relationship offers a general characterization of how competition in the mutual funds industry will induce an equilibrium pricing of risk in a way which is familiar from the capital asset pricing model (CAPM). It should, however, be emphasized that in our model the relationship between expected return and risk comes about in a world with imperfectly competitive mutual funds rather than a world with perfect competition where investors individually construct efficient portfolios from available assets. Below we formally characterize the equilibrium relationship between expected return and risk.

As before, let m_i denote the share of fund i's portfolio invested into the risky asset. In view of (1), the random return on each dollar invested into fund i is

$$R_i = \begin{cases} m_i\alpha + (1 - m_i)\beta & \text{with probability } \theta, \\ (1 - m_i)\beta & \text{with probability } 1 - \theta \end{cases}$$

so that the expected return is

$$\mathrm{E}R_i = \theta m_i \alpha + (1 - m_i)\beta. \tag{22}$$

Therefore, the variance of fund i is given by

$$\sigma_i^2 = \mathrm{E}[R_i - \mathrm{E}R_i]^2 = \theta(1 - \theta)\alpha(m_i)^2,$$

so that

$$m_i = \frac{\sigma}{\alpha\sqrt{\theta(1 - \theta)}}. \tag{23}$$

Substituting (23) for m_i into (22) yields a version of the mean-variance relationship, or more precisely, the expected return as a function of the standard deviation. Thus,

$$\mathrm{E}R_i(\sigma) = \beta + \frac{(\theta\alpha - \beta)\sigma}{\alpha\sqrt{\theta(1 - \theta)}}. \tag{24}$$

From (24) we observe that

$$\mathrm{E}R_i(0) = \beta \quad \text{and} \quad \frac{\mathrm{d}\mathrm{E}R_i(\sigma)}{\mathrm{d}\sigma} = \frac{\theta\alpha - \beta}{\alpha\sqrt{\theta(1 - \theta)}} > 0.$$

Fig. 3 illustrates the relationship between expected return on mutual funds and their standard deviation around the mean return.

When placing (24) into a standard CAPM-framework we can infer that the traditional β-factor, i.e., the slope to the security market line, is given by $(\theta\alpha - \beta)/[\alpha\sqrt{\theta(1 - \theta)}]$. Finally, Fig. 3 provides a nice interpretation of our results in Section 4. Section 4 shows that fund 2 will diversify its portfolio so that the standard deviation of its return will satisfy $\sigma_2 = (\sigma_3 - \sigma_1)/2$, which is the average standard deviation of the two competing funds. However, the

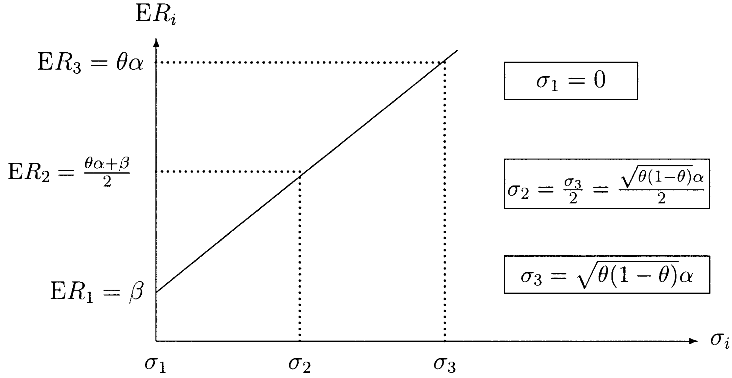

Fig. 3. Mean–SD relationship in the market for mutual funds.

expected return of the intermediate fund is more than one-half of the risky fund, i.e.,

$$\frac{\theta\alpha}{2} = \frac{\mathrm{E}R_3}{2} < \mathrm{E}R_2 = \frac{1}{2}\theta\alpha + \frac{1}{2}\beta < \theta\alpha = \mathrm{E}R_3, \quad \text{since } \theta\alpha > \beta > 0.$$

6. Conclusion

In light of the explosive growth of mutual funds available to investors it is extremely important to investigate the effects of market structure in the mutual funds industry on the investment behavior and risk taking of these funds. Our analysis presents a general characterization of how imperfect competition among investment funds differentiated by risk will generate an equilibrium relationship between risk and expected returns of these funds.

The present study offers a first, pioneering attempt at addressing this general issue. From our analysis we are able to characterize a systematic relationship between the diversification incentives and the market structure of the mutual funds industry. With sufficiently low competition, i.e., in a duopoly, the subgame perfect portfolio equilibrium will exhibit maximal risk differentiation, because the Nash equilibrium fees are linearly increasing functions of the risk differential. With intensified competition we find that intermediate funds, i.e., those attracting investors with intermediate attitudes towards risk, will select diversified portfolios.

Also our analysis has made clear that the portfolio equilibrium is not characterized by rent equalization between the funds. Instead, as the case with three competing funds has shown, the subgame perfect profits are asymmetric.

Such an asymmetry typically has interesting implications for the pattern of industry entry and exit. The implications of such a phenomenon have been studied in models of vertical product differentiation by, for example, Shaked and Sutton (1982, 1983) and Anderson et al. (1992). It could be an interesting direction for further research to investigate the industry dynamics generated by entry and exit for industries characterized by risk differentiation like the present model.

Of course, the model presented has abstracted from a number of important considerations. The model represents a partial equilibrium analysis where we have excluded investment opportunities outside of the mutual funds industry. An initial step to incorporate generalizations in such a direction would be to endow the investors with outside options the values of which are determined by the risk-return combinations offered by alternative financial assets. Such a generalization could be particularly interesting insofar as it could offer insights into whether the mutual funds industry is likely to face tougher competition from alternative assets at the upper or lower end of the risk spectrum. Further, our analysis has abstracted from agency issues associated with informational advantages of fund managers relative to investors. Inclusion of such aspects into the question of how increased competition between mutual funds affects the incentives for diversification would require a, still missing, general theory of competing pairs of principals and agents within the context of imperfectly competitive markets.

Appendix A. Diversifying investors

Our analysis relied on the assumption that investors choose one and only one fund to invest in. In this appendix, we demonstrate that risk-neutral investors indeed choose only one fund even if they are allowed to diversify their saving among all available funds.

Suppose now that an investor of type λ ($0 \leqslant \lambda \leqslant 1$) can diversify the savings without any restrictions. Let w_λ^1 and w_λ^2 denote the weights investor type λ assigns to funds 1 and 2, respectively, where $w_1, w_2 \geqslant 0$ and $w_1 + w_2 = 1$. Then, in view of (2), investor type λ chooses w_1 and w_2 to solve

$$\max_{w_1,w_2} \mathrm{E}_\lambda R(w_1, w_2) = w_1 \mathrm{E}_\lambda R_1 + w_2\, \mathrm{E}_\lambda R_2. \tag{A.1}$$

Notice that the expected return depends on λ (probability of a liquidity need of a particular investor). However, due to risk neutrality of investors, (A.1) is linear in the portfolio's weights. Since $w_1 = 1 - w_2$, the solution to (A.1) is

$$\langle w_1, w_2 \rangle = \begin{cases} \langle 1, 0 \rangle & \text{if } \mathrm{E}_\lambda R_1 \geqslant \mathrm{E}_\lambda R_2, \\ \langle 0, 1 \rangle & \text{if } \mathrm{E}_\lambda R_1 < \mathrm{E}_\lambda R_2. \end{cases}$$

References

Allen, F., Gale, D., 1994. Financial Innovation and Risk Sharing. The MIT Press, Cambridge, MA.

Anderson, S., de Palma, A., Thisse, J., 1992. Discrete Choice Theory of Product Differentiation. The MIT Press, Cambridge, MA.

Broecker, T., 1990. Credit-worthiness tests and interbank competition. Econometrica 58, 429–452.

Chordia, T., 1996. The structure of mutual fund charges. Journal of Financial Economics 41, 3–39.

Dybvig, P., Farnsworth, H., Carpenter, J., 1999. Portfolio performance and agency, Working paper, Washington University, St Louis.

Gehrig, T., 1998. Screening, cross-border banking and the allocation of credit. Research in Economics 52, 387–407.

Grinblatt, M., Titman, S., Wermers, R., 1995. Momentum investment strategies, portfolio performance, and herding: a study of mutual fund behavior. American Economic Review 85, 1088–1105.

Khorana, A., Nelling, E., 1998. The determinants and predictive ability of mutual fund ratings. Journal of Investing 7, 61–67.

Koskela, E., Stenbacka, R., 2000. Is there a tradeoff between bank competition and financial fragility? Journal of Banking and Finance 24, 1853–1873.

Massa, M., 2000. Why so many mutual funds? Mutual funds, market segmentation and financial performance, Working paper, INSEAD.

Matutes, C., Vives, X., 2000. Imperfect competition, risk taking, and regulation in banking. European Economic Review 44, 1–34.

Nanda, V., Narayanan, M.P., Warther, V., 2000. Liquidity, investment ability, and mutual fund structure. Journal of Financial Economics 57, 417–443.

Shaffer, S., 1998. The winner's curse in banking. Journal of Financial Intermediation 7, 359–392.

Shaked, A., Sutton, J., 1982. Relaxing price competition through product differentiation. Review of Economic Studies 49, 3–13.

Shaked, A., Sutton, J., 1983. Natural oligopolies. Econometrica 51, 1469–1483.